M225: Linear Algebra and Linear Differential Equations
Summer 2022

Instructor: Dmitrii M. Ostrovskii (dostrovs@usc.edu), KAP 406h
Instructor's office hours: Wed 1:15-3:00 pm
TA: Gin Park (ginpark@usc.edu), KAP 262a
TA's office hours: Tue 9:30-10:20 am, GFS 207
Grader: Zejing Wang (zejingwa@usc.edu) KAP 263 (Math Center)
Schedule: M/Tue/W/Thu/F 10:30-12:20 am, GFS 207 (discussions on Tue/Thu)
Zoom meeting ID: 951 4105 6073

Disclaimer. *This syllabus is not a contract: the Instructor has the right to make changes at his discretion throughout the semester. However, these changes are not likely to be significant.*

About the course. The goal of the first part of the course is to give a solid—even though somewhat fast-paced—introduction to linear algebra. Building upon this foundation, in the second part of the course we shall focus on linear differential equations (LDEs) and systems of LDEs, and look at least-squares linear regression if time permits. The main textbook is **Goode and Annin – Differential Equations & Linear Algebra (4th ed.)**, Chap. 2-7 (entirely) and selected parts of Chap. 8-9; it can be purchased from the USC bookstore. Other references:

- MIT OCW videolectures on Linear Algebra by Prof. Gilbert Strang. Great exposition and some topics beyond the curriculum (e.g. application to graphs and fast Fourier transform).
- App. A.5 (except A.5.5) and C.1-C.3 of Boyd and Vanderberghe – Convex Optimization is a "cheat sheet" rather than a textbook, especially helpful in the later part of the course.

Scribblers shall also be assigned on a rotating basis beginning from the 3rd instruction week.

Prerequisites: M126, M127 or M129—in other words, some version of Calculus II. The very basics of complex analysis—on the level of multiplying complex numbers—will be useful in the latter part of the course, but shall be covered in class if the need arises.

Course outline:

1. Solving $Ax = b$ with a square A via the Gaussian elimination method.
2. LU-decomposition; the Gauss-Jordan method. Inverse and transpose.
3. Vector spaces and subspaces. Column space and nullspace of a matrix.
4. General solution for homogeneous systems $Ax = 0$ with a rectangular matrix.
5. Linear independence. Basis and dimension of a vector space. Matrix rank.
6. Solvability of $Ax = b$ (Rouché–Capelli theorem). General solution method.

7. Change of basis. Linear transformations. Singular value decomposition (SVD).
8. Eigenvalue problem. Determinant. Characteristic equation.
9. Euclidean spaces; self-adjoint operators. Eigenvalue decomposition of a symmetric matrix.
10. Projector on a subspace. Pseudoinverse. Application to least-squares linear regression.
11. Jordan canonical form (JCF) of a matrix.
12. LDEs and shift-invariant subspaces. General solution of a homogeneous LDE via JCF.
13. Matrix exponent and systems of LDEs.
14. Solving nonhomogeneous (systems of) LDEs via the method of variation of constants.

Contact and questions. The best way to ask a question is in class after the lecture. The second best way, especially with "private" matters—is by email: put **M225** into the subject field). I do my best to respond within 24 hours, but sometimes it takes longer. I generally **do not** reply on weekends (or at least try not to). Also, you may come in in the office hours; prior notice is not required but appreciated. Please **knock on the door** before coming in. Email me to book an appointment outside of the office hours if you have a scheduling conflict with them.

Homework. There will be 5 or 6 weekly homework assignments, posted on blackboard (BB).

- Always start to work on homework assignments as early as possible.
- It's okay to work in a small group. However, always try on your own first. If you are blocked, take a walk, go to the beach, etc., and then think again. If still blocked after 2 attempts, use TA's and/or Instructor's office hours. Please take notes during such sessions!
- Please make sure what you wrote is readable. Always write the solution on your own; suspected plagiatrism will receive **0 credit** and further sanctions in case of a relapse.
- **Please submit your homework solutions on Blackboard in a PDF file.** Pages must be numbered. One way is to scan your handwritten solution, use iPad + iPencil + a handwriting app such as Notability. Alternatively, typeset your solution in LaTeX; this is especially recommended if you're planning to pursue a PhD in a quantitative field.

Conduct code. You can collaborate on homeworks, but you must **write solutions on your own**. Suspected cases of plagiatrism will be (ruthlessly) sanctioned as per the USC Student Conduct Code; see https://dornsife.usc.edu/usc-policies/#plagiarism. Cheating on an exam nullifies the score for that exam. **All** personal devices must be **turned off** on the exam.

Exams schedule. There will be 1 (one) midterm and a final exam. The midterm will be 75 minutes long and take place in our regular classroom (GFS 207) some day between June 15 and June 22, in our regular hours. The 2-hour long final exam will be scheduled for the early July.

Grading breakdown: quizzes (13%), homework (27%), midterms (25%), final exam (35%). Please note that the midterm and final exams are **compulsory** to attend. To hedge against non-attendance and emergencies, I will drop one homework and one quiz with the lowest score.

Helpful resources. The Math Center (KAP 263) is open Mon-Fri 8am-7pm on most days. It is primarily run by math graduate students. You may also access the virtual Math Center zoom room at https://usc.zoom.us/j/97118086999. For more information on the Math Center, see https://dornsife.usc.edu/mathcenter/. Other resources:

- For help with Blackboard see https://itservices.usc.edu/blackboard/.
- For the registration schedule see https://classes.usc.edu/term-20222/classes/math/.
- For student support resources (psychological support, bias/harassment prevention, etc.) see https://academicsenate.usc.edu/resources/student-resources-and-support/.

Attendance. I shall rely on your motivation and maturity, and not keep track of attendance. I will generally warn about a quiz in advance, but I do not commit on always doing so. My recommendation is to attend as much as possible to get a broader perspective on the material.