Adaptive recovery of signals by convex optimization

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jointly with

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Estimate x_n given n regular samples of signal $x \in \mathcal{X}$ with $\xi_t \sim \mathcal{N}(0,1)$

$$y_t = x_t + \sigma\xi_t, \quad t = 1, \ldots, n.$$

- Linear estimate of x_n is given by a **filter** $\varphi \in \mathbb{R}^n$ as $\widehat{x}_n = [\varphi * y]_n$
- If $\mathcal{X}$ is convex, linear estimates enjoy **minimaxity** (classical result)

How to adapt to the unknown minimax linear estimator?

Main assumption

There exists a time-invariant filtering which recovers the last $O(n)$ samples with error $O_{\mathbb{P}}(1/\sqrt{n})$.

$(A\rho)$ For each $x \in \mathcal{X}$ there exists $\varphi^{\text{oracle}} \in \mathbb{R}^{n/4}$ such that

$$\left\|\varphi^{\text{oracle}}\right\|_2 \le \frac{\rho}{\sqrt{n}},$$

$$\left|\left[x - \varphi^{\text{oracle}} * y\right]_t\right| \le \sigma\, O_{\mathbb{P}}\left(\frac{\rho}{\sqrt{n}}\right), \quad \frac{n}{4} \le t \le n.$$

Lower bound

For any $n \in \mathbb{Z}_+$ and $\rho \ge 0$, there exists a family of signals $\mathcal{F}_n(\rho)$ satisfying **$(A\rho)$**, and such that

$$\inf_{\widehat{x}_n} \sup_{x \in \mathcal{F}_n(\rho)} \mathbb{E}^{1/2}\left(\widehat{x}_n - x_n\right)^2 \ge C\,\sigma\,\frac{\rho^2\sqrt{\ln n}}{\sqrt{n}}$$

Estimator

For any $x \in \mathbb{R}^n$ let $\|x\|_p^* := \|\mathrm{DFT}(x)\|_p$

For desired confidence level α, find a solution $(\widehat{\varphi}, \widehat{R})$ of

$$\min_{\varphi, R} R \quad \text{subject to}$$

$$\varphi \in \mathbb{C}^{n/2}, \quad \|\varphi\|_1^* \leq R\sqrt{\tfrac{2}{n}},$$
$$\left\| [y - \varphi * y]_{n/2}^n \right\|_\infty^* \leq 2\sigma(R+1)\sqrt{\ln\left(\tfrac{n}{2\alpha}\right)}$$

Then build the estimate

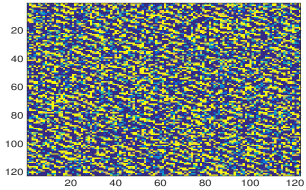
$$\widehat{x}_n = [\widehat{\varphi} * y]_n$$

- Filter-fitting problem is well-structured second-order cone problem (all ℓ_p-norms are of complex vectors)
- Enjoys minimax rate (in terms of n but not ρ)

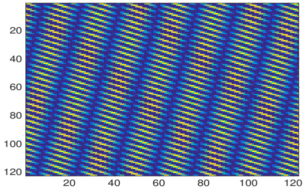
Numerical results

Prediction of a 2-d signal – sum of 2 sinusoids with unit amplitudes and random frequencies. SNR $= -3\,$dB, $n = 12$.

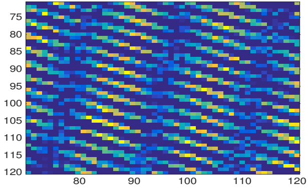
a) noisy signal



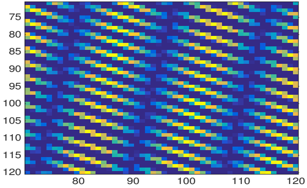
b) true signal



c) prediction in the target zone



d) true signal in the target zone



Thanks and see you at the poster session!