Math 7252: High-Dimensional Statistics

Fall 2025
Schedule & location: M/W 2pm–3:15pm, Skiles 269
Instructor: Dmitrii M. Ostrovskii – ostrov@gatech.edu – Skiles 263
Office hours: M/W 4pm–5:30pm, Skiles 269 + on request by email
Canvas: https://gatech.instructure.com/courses/480610

Disclaimer. This course is targeted primarily at graduate students whose research interests are in statistical theory and mathematics of data, as well as at the intersection of these with information theory and optimization. Along with classical topics—concentration inequalities and their applications in high-dimensional estimation, covariance matrix estimation, sparse recovery, compressive sensing—we will cover some very modern developments: estimation under structural constraints, modern robust estimation, quantum tomography. Some open problems will be introduced and discussed. Some of the above will be offered as final presentation topics.

Prerequisites. Probability at the level of Math 6241 is preferable, but not required. There is a some overlap of topics with Math 7251, but this class provides a different outlook on these.

Literature. Vershynin's monograph [Ver18] covers the bulk of the first 7 topics. In addition, my ISyE 8803 lecture notes ([LN] below), shared on Canvas, will be heavily used in the first weeks, and occasionally later. Some dedicated references for separate topics are provided below.

Outline of topics. Mileage may vary, so the list below is approximate – but representative.

- Probability recap. Chernoff's method. Gaussian tail bounds and Stein's phenomenon [LN].
- Subgaussian random variables, their sums and maxima [LN], [Ver12].
- Subexponential distributions. Bernstein's inequality, ℓ_2-norm of a Gaussian vector [Ver12].
  - Applications: Johnson-Lindenstrauss lemma, Grothendieck inequality & MAX-CUT.
- Estimation of covariance and Gram matrices [Ver12].
  - Applications: random-design linear regression [HKZ12], Newton's method [TBA], graph sparsification [SS08], community detection [Ver18].
- Matrix Bernstein [Tro15, Ver18].
- Symmetrization and decoupling [Ver18].
  - Applications: U-statistics [DlPG12, Chapter 3], estimation of moment tensors [LN].
- Stochastic processes. Gordon, Slepian and Sudakov. Dudley's chaining method [Ver18].
- Robust estimation of mean & covariance [Min15, WM17]. Equiaffine estimators [OR19].
- Near-optimal estimator of Lugosi-Mendelson [LM19] and tractable version [Hop20].
- Sparse recovery in direct observations (denoising) and linear measurements [CRPW12].
- Low-rank matrix recovery [CRPW12]. Quantum tomography [TBA].
- "Conventional" super-resolution, à-la Candès [CFG13] or Tang-Bhaskar-Recht [TBR14].
- Estimation under shift-invariance, or "super-resolution beyond the Abbe limit" [Ost24].
  - Open problem: Deconvolution under shift-invariance.

Contact and office hours. The best way to contact me is by email (please add "Math 7252" to the subject line), in person in the office hours, or by appointment (better in the afternoon).

Grading. 20% × 4 homework assignments; 20% final presentation (topics TBA in November).

Attendance. In general, I do not keep track of attendance. However, if attendance drops below a certain point, I reserve the right to start controlling it with modest bonuses and penalties.

References

[CFG13] E. J. Candès and C. Fernandez-Granda. Super-resolution from noisy data. *Journal of Fourier Analysis and Applications*, 19(6):1229–1254, 2013.

[CRPW12] V. Chandrasekaran, B. Recht, P. A. Parrilo, and A. S. Willsky. The convex geometry of linear inverse problems. *Foundations of Computational mathematics*, 12(6):805–849, 2012.

[DlPG12] V. De la Pena and E. Giné. *Decoupling: from dependence to independence*. Springer Science & Business Media, 2012.

[HKZ12] D. Hsu, S. M. Kakade, and T. Zhang. Random design analysis of ridge regression. *The Journal of Machine Learning Research*, 23(9):1–24, 2012.

[Hop20] S. B. Hopkins. Mean estimation with sub-gaussian rates in polynomial time. *The Annals of Statistics*, 48(2):1193–1213, 2020.

[LM19] G. Lugosi and S. Mendelson. Sub-gaussian estimators of the mean of a random vector. *Annals of Statistics*, 47(2):783–794, 2019.

[Min15] S. Minsker. Geometric median and robust estimation in Banach spaces. *Bernoulli*, 21(4):2308–2335, 11 2015.

[OR19] D. M. Ostrovskii and A. Rudi. Affine invariant covariance estimation for heavy-tailed distributions. In *Proceedings of the Thirty-Second Conference on Learning Theory*, volume 99, pages 2531–2550, Phoenix, USA, 25–28 Jun 2019. PMLR.

[Ost24] D. M. Ostrovskii. Near-optimal and tractable estimation under shift-invariance. *arXiv preprint arXiv:2411.03383*, 2024.

[SS08] D. A. Spielman and N. Srivastava. Graph sparsification by effective resistances. In *Proceedings of the fortieth annual ACM symposium on Theory of computing*, pages 563–568, 2008.

[TBR14] G. Tang, B. N. Bhaskar, and B. Recht. Near minimax line spectral estimation. *IEEE Transactions on Information Theory*, 61(1):499–512, 2014.

[Tro15] J. A. Tropp. An introduction to matrix concentration inequalities. *Foundations and Trends® in Machine Learning*, 8(1-2):1–230, 2015.

[Ver12] R. Vershynin. Introduction to the non-asymptotic analysis of random matrices. In *Compressed Sensing: Theory and Applications*, pages 210–268. Cambridge University Press, 2012.

[Ver18] R. Vershynin. *High-dimensional probability: An introduction with applications in data science*, volume 47. Cambridge university press, 2018.

[WM17] X. Wei and S. Minsker. Estimation of the covariance structure of heavy-tailed distributions. In *Advances in Neural Information Processing Systems*, pages 2859–2868, 2017.